Supplementary Material

The Reynolds equation can be written in dimensionless form:

$$\frac{\partial^2 \tilde{p}}{\partial a^2} + \frac{\partial^2 \tilde{p}}{\partial b^2} + \frac{6a}{\tilde{h}}\frac{\partial \tilde{p}}{\partial a} + \frac{6b}{\tilde{h}}\frac{\partial \tilde{p}}{\partial b} = \frac{6a}{\tilde{h}^3} \qquad \text{(A1)}$$

where the dimensionless coordinates a and b, the dimensionless height h and the dimensionless pressure $\tilde{p}$ are defined as:

$$a^2 \equiv \frac{x^2}{2Rh_0} \qquad b^2 \equiv \frac{y^2}{2Rh_0} \qquad \tilde{h} \equiv \frac{h}{h_0} = 1 + a^2 + b^2 \qquad \tilde{p} \equiv \frac{p}{p_0} \qquad \text{(A2a-d)}$$

with

$$p_0 \equiv \frac{2\sqrt{2Rh_0}\eta U_0}{h_0^2} \qquad \text{(A3)}$$

In these coordinates, there is one dimensionless parameter describing the size of the dimensionless domain:

$$\widetilde{R}_s = \frac{R_s}{\sqrt{2Rh_0}} \qquad \text{(A4)}$$

It is imposed that, at the boundary of the domain, the pressure must be zero. Thus, the solution for the pressure depends on $\tilde{R}_s$. We define a tilt parameter f, where $f = 0\%$ means this origin is positioned in the center, and $f = 100\%$ means it is positioned at the edge of the domain. With these conditions, Eq. (A1) is solved numerically using Mathematica. With the pressure distribution $\tilde{p}(a, b)$ calculated, the normal force is found by integrating the pressure distribution over the entire surface area, whereas the friction is found by integrating the shear stress τ_{xz} in x-direction at $z = h$ over the entire surface area. The shear stress is calculated by $\tau_{xz} = \eta \frac{\partial u_x}{\partial z}$, where the velocity profile u_x is given in the lubrication approximation. Because a negative pressure is unphysical, cavitation will occur in the outlet region where $\tilde{p} < 0$ [1]. Following [1], this region should not be included into the integral calculating the lift.

The lift, which in equilibrium is equal to the normal force, and the friction are then given by:

$$F_N = \int_D p\, dxdy = \frac{8\eta U_0 R^2}{R_s} \cdot \Lambda(\widetilde{R}_s) \qquad \text{(A5)}$$

$$F_w = \int_D \tau_{xz}\, dxdy = 2\eta U_0 R \cdot K(\widetilde{R}_s) \qquad \text{(A6)}$$

where the following dimensionless integrals are defined:

$$\Lambda(\widetilde{R}_s) \equiv \widetilde{R}_s \int_{\widetilde{D}} \tilde{p}\, dadb \qquad \text{(A7)}$$

$$K(\widetilde{R}_s) \equiv \int_{\widetilde{D}} \left(\frac{1}{\tilde{h}} + \tilde{h}\frac{\partial \tilde{p}}{\partial a}\right) dadb \qquad \text{(A8)}$$

Here, the dimensionless domain $\widetilde{D}$ has radius $\tilde{R}_s$ and has its center shifted along the a-axis by $f \cdot \tilde{R}_s$ units. Note that, though the integrals are dimensionless, they do still depend on the h_0, R, Rs and f through this domain. Kapitza [2] and Brewe *et al.* [3] have analyzed this problem in a similar fashion, but by considering a sliding sphere instead of a disk, effectively taking the limit $\tilde{R}_s \gg 1$. Therefore, their equivalent of Eq. (A5) only contains a dimensionless constant determined by numerical simulations, instead of a dimensionless function. The natural tendency of the disk to tilt is a result of the pressure distribution being asymmetric, and it turns out to be determined by the dimensionless parameters only. Taking $a = 0$ as point of reference, the pressure distribution creates a counterclockwise torque-element $\tilde{p} \cdot a\, dadb$ at each point $(a,\ b)$ on the domain. Assuming equilibrium condition, this torque balanced by the normal force placed centrally on the disk at position $f{\cdot}\tilde{R}_s$, hence the following relation, which relates $\tilde{R}_s$ and f, should hold:

$$\int_{\widetilde{D}} \widetilde{p} \cdot a\, dadb = f \cdot \widetilde{R}_s \cdot \int_{\widetilde{D}} \widetilde{p}\, dadb \quad \text{(A9)}$$

The solution of this equation is plotted in Fig. A1a and used throughout the numerical calculation of $\Lambda(\tilde{R}_s)$ and $K(\tilde{R}_s)$, which are shown in Fig. A1b.

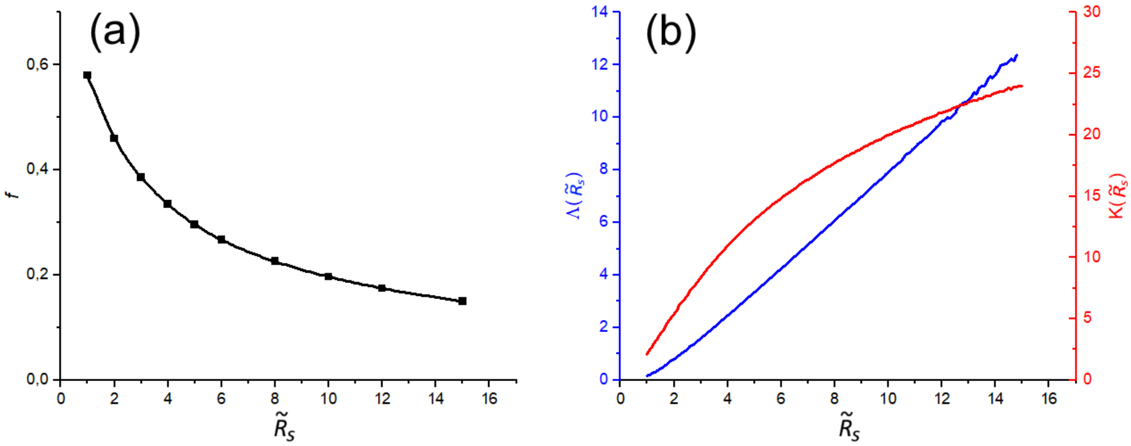

Figure A1: (a) Solution for f as a function of $\tilde{R}_s$ found by numerically solving Eq. (A9). As a guide to the eye, data points are connected by a spline (solid line). (b) Dimensionless functions $\Lambda(\tilde{R}_s)$ (blue line) and $K(\tilde{R}_s)$ (red line) as found numerically from Eqs (A7) and (A8). Non-smoothness at high $\tilde{R}_s$ is an artefact of numerical limits.

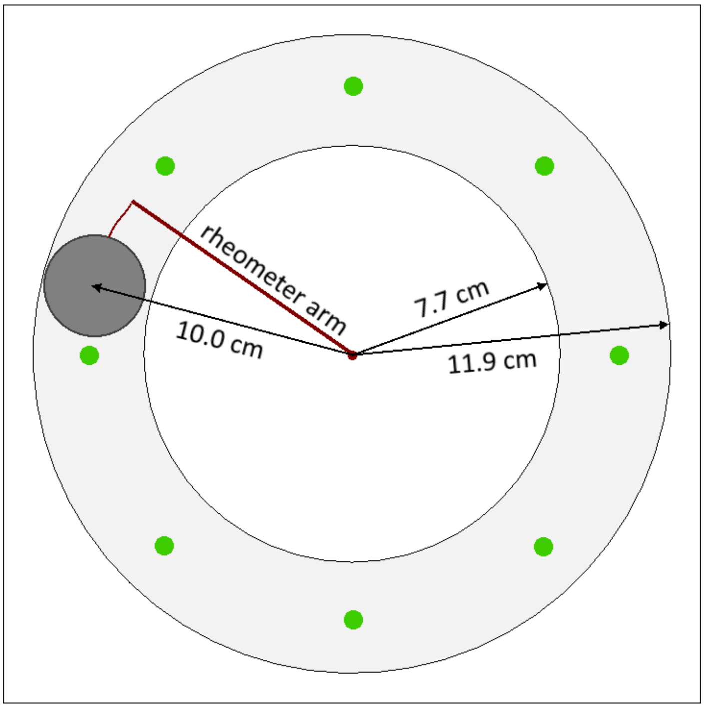

Figure A2: Top view of the disk moving over the circular glass track (light grey area) confined between two walls (white areas). The green dots represent the inductive sensors glued underneath the glass at regular 45° intervals. The disk is attached by a flexible thin wire (brown curvy line) to a rotating rheometer arm (brown straight line).

[1] C.H. Venner and A. Lubrecht. Multilevel methods in lubrication. (Elsevier, Amsterdam, 2000)
[2] P.L. Kapitza. The Hydrodynamic Theory of Lubrication In The Presence Of Rolling. Zh. Tekh. Fiz. **25**, 747-762 (1955).
[3] D.E. Brewe, B.J. Hamrock and C.M. Taylor. Effect of Geometry on Hydrodynamic Film Thickness, J. Lubr. Technol. **101**, 231-239 (1979)